

Time and Space Complexity

for (i=1; i<=n; i++) 1...n
 { print(i); → O(n)

for (i=1; i<n; i++) → O(n-1)
 { print(i); n = 10⁶ ⇒ O(n)
 n = 10⁵ →

O(n+1) → O(n)

if (x%2==0) → O(1)
 print("even"); constant

for (i=1; i<=n; i=i+2) 1, 3, 5, 7, ... n

{
}

$n = \underline{10^6}$ → $O(n/2)$
↳ $O(n)$
 $\frac{n}{2} = 5 \text{ lakh} = \underline{5 \times 10^5}$

for (i = 2; i <= n; i += 2)

2, 4, 6...

{
 print(i);
}

→ $O(n/2) \rightarrow O(n)$

$i \leq \sqrt{n}$

for (i = 1; $i \times i \leq n$; i++)

→ $O(\sqrt{n})$

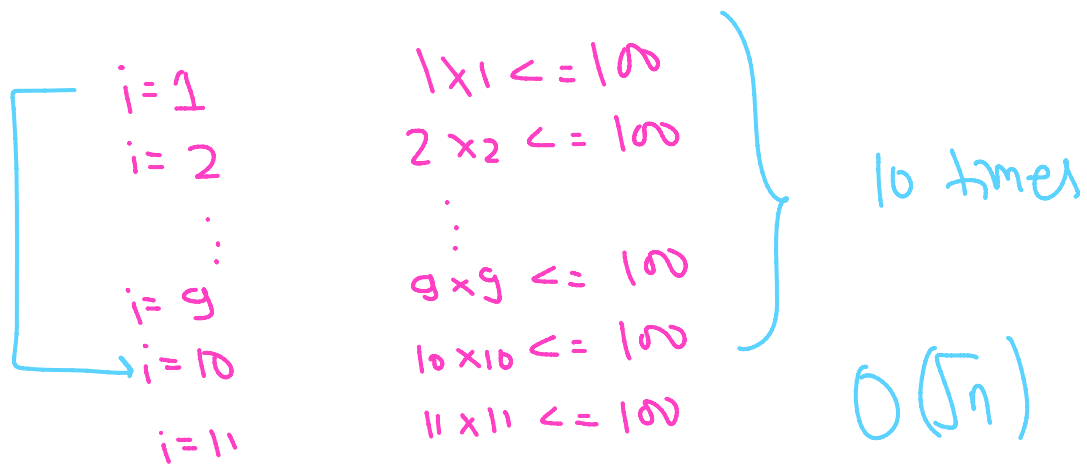
{
 print(i);
}

$i^2 \leq n$

$i \leq \sqrt{n}$

$O(\sqrt{n})$

$n = 100 \Leftarrow$



$n = 10^6$
 10 Lakh

$\sqrt{n} \rightarrow 10^3$
 1000 = 1 thousand

for (i=1; i*i*i <= n; i++)

{
 print(i);
 }

$O(\sqrt[3]{n})$

(cube root of n)

for (i=1; i <= n; i++)

{
 break;
 }

$\rightarrow O(1)$

for (i=1; i<=n; i=i*2)

{
 print(i);
}
→ O(n)

n = 70

i = 1, 2, 4, 8, 16, 32, 64, 128
 ↓ ↓ ↓ ↑
 2⁰ 2¹ 2²

i = 1, 2, 4, 8, 16, ..., n
 ↓ ↓ ↓ ↓
→ 2⁰, 2¹, 2², ..., 2^k ⇒ O(k+1)
⇒

$$n = 2^k$$

$$2^k = n$$

take \log_2 on both sides

$\log_a b = (x) \log_a c$

$$\log_2 2^k = \log_2 n$$

..... 1 n

$$\log_a b = (\log_a b)^{\cdot}$$

$$k \times \log_2 2 = \log_2 n$$

$$\log_a a = 1$$

$$k \times 1 = \log_2 n$$

$$k = \log_2 n$$

$$= O(k+1)$$

$$= O(k)$$

$$\Rightarrow O(\log_2 n)$$

for (i=1; i<=n; i=i*3)

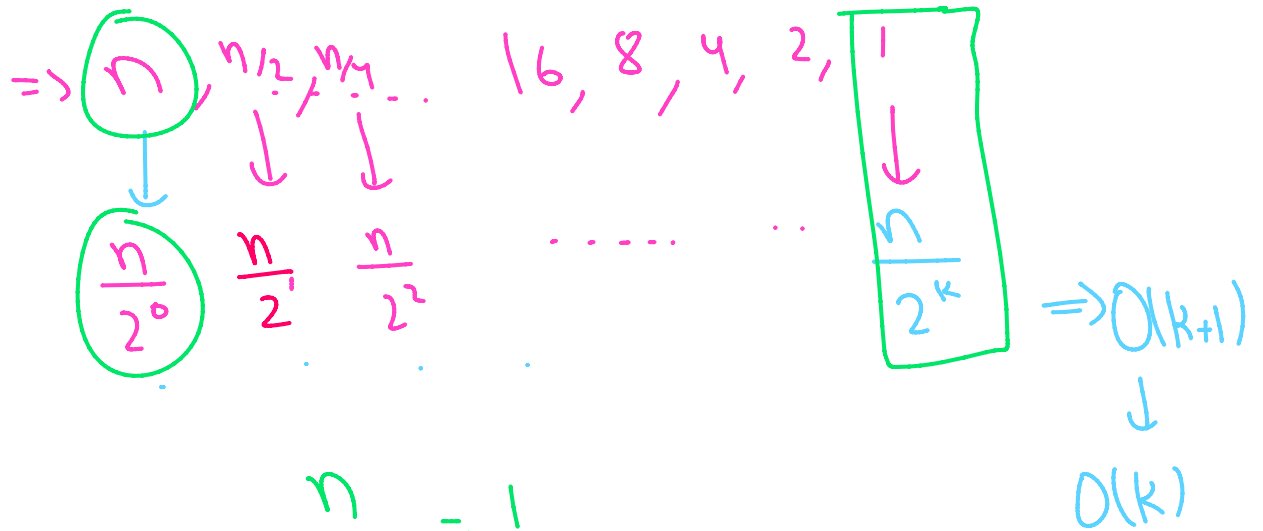
{
 print();
}

$$\Rightarrow O(\log_3 n)$$

for (i=n; i>=1; i=i/2)

{
 print();
}

$\Rightarrow 64, 32, 16, 8, 4, 2, 1$



$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

take \log_2

$$\log_2 2^k = \log_2 n$$

$$k \times \log_2 2 = \log_2 n$$

$$k = \log_2 n$$

$$\rightarrow O(k)$$

$$\rightarrow O(\log_2 n)$$

```
for (i=n; i=1; i=i/x)
```

```
{  
  print();  
}
```

$O(\log_x n)$

```
for (i=1; i<=n; i++)
```

```
{  
  for (j=1; j<=n; j++)  
  {  
    print();  
  }  
}
```

i=1 i=2 i=3 i=n
ntimes ntimes ntimes ... ntimes

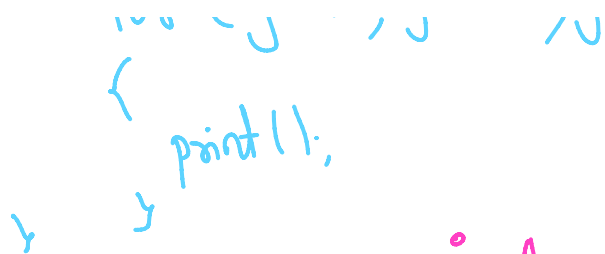
$$3+3+3=9=3^2$$

$$4+4+4+4=16=4^2$$

$$\underbrace{n+n+\dots+n}_{ntimes} = n^2$$

```
for (i=1; i<=n; i++)
```

```
{  
  for (j=1; j<=i; j++)  
  {  
    ...  
  }  
}
```



$i=1$ $i=2$ $i=3$ $i=4$... $i=n$
 1 times 2 times 3 times 4 times ... n times

$$\Rightarrow 1 + 2 + 3 + \dots + n$$

\Rightarrow Sum of n natural nos

$$= \frac{n(n+1)}{2}$$

$$O(n)$$

$$O(\sqrt{n})$$

$$O(n/2)$$

$$O(n^2)$$

$$x^6 + 8x^5 + 10x^4 + 3x^2 + 4x + 7 = 0$$

\hookrightarrow degree = 6

$$= \frac{n(n+1)}{2} = \frac{n^2 + n}{2}$$

$$\Rightarrow O\left(\frac{n^2}{2} + \frac{n}{2}\right)$$

$$n = 10^3$$

$$\Rightarrow O(n^2 + n)$$

$$\begin{array}{c} \downarrow \quad \downarrow \\ 10^6 + 10^3 \end{array}$$

$$= O(n^2) + O(n)$$

$$\Rightarrow O(n^2)$$

```
for (i=1; i<=n; i++)  
{  
  for (j=1; j<=i; j++)  
  {  
    print();  
  }  
}
```

$i=1$ $i=2$ $i=3$ $i=n$
0 times 1 time 2 times $n-1$

$$\Rightarrow 0 + 1 + 2 + \dots + n-1 \quad O(n^2)$$

$$1 + 2 + 3 + \dots + P \Rightarrow \frac{P(P+1)}{2}$$

$$\frac{(n-1)(n-1+1)}{2} \Rightarrow \frac{(n-1)(n)}{2}$$

```
for (i=1; i<=n; i++)  
{
```

```
  for (j=1; j<=n; j=j*2)  
  {
```

```

100 ( J - 1 / J . . . / J J .
{
  print();
}
}

```

$i=1$ $i=2$ $i=3$ \dots $i=n$
 \log_2^n \log_2^n \log_2^n \dots \log_2^n

$\Rightarrow \log_2^n + \log_2^n + \dots + \log_2^n$
└──────────────────────────┘
n times

$2+2+2 = 6$
 $= 3 \times 2$

$4+4+4 = 12$
 $= 3 \times 4$

$\Rightarrow n \log_2^n$

```

for ( i=1; i<=n ; i++)

```

```

{
  for ( j=n; j>=1 ; j=j/2)

```

```

{
  print();
}
}

```

$i=1$ $i=2$ \dots $i=n$
 \log_2^n \log_2^n \dots \log_2^n

$\Rightarrow n \log n$

```
for (i=1; i<=n; i++)
```

```
{
```

```
  for (j=1; j<=n; j++)
```

```
  {
```

```
    if (j%2==1)
```

```
    { break; }
```

```
  } }
```

$i=1$ $i=2$ $i=3$... $i=n$
1 time 1 time 1 time ... 1 time

$= \underbrace{1+1+\dots+1}_{n \text{ times}} \Rightarrow n \text{ times}$

$O(n)$

```
for (i=1; i <= n; i++)
```

```
{
```

```
  for (j=1; j=2 j<=n; j++)
```

```
  {
```

```
    if (j%2==0)
```

```
    { break; }
```

...n

$$1+2+\dots+p \leq n$$

$$\frac{p(p+1)}{2} = n$$

$$p^2 + p = 2n$$

$$p^2 + p - 2n = 0$$

$$ap^2 + ap - 2n = 0$$

Q. for $(i=0; i \leq N; i++) \rightarrow O(N)$

{

$$a = a + \text{rand}();$$

}

for $(j=0; j \leq M; j++) \rightarrow O(M)$

{

$$b = b + \text{rand}();$$

}

$$TC \Rightarrow O(N+M)$$

$$Sc: O(1)$$

```

for (i=0; i<N; i++)
{
    for (j=N; j>i; j--)

```

```

    {
        a = a + i * j;
    }
}

```

$i=0$ $i=1$ $i=2$... $i=N-1$
 N times $N-1$ times $N-2$... 1

$$N + (N-1) + \dots + 1$$

$$\Rightarrow 1 + 2 + \dots + (N-1) + N$$

$$= \frac{N(N+1)}{2} \rightarrow O(N^2)$$

```

for (i=n/2; i<=n; i++)

```

```

{
    for (j=2; j<=n; j=j*2)

```

```

    {
        k = k + n/2;
    }
}

```

$i=n/2$ $i=n/2+1$ $i=n$
 $\log n$

$\}$ $i = n/2$ $i = \lceil n/2 \rceil$ \dots
 $\log_2 n$ $\log_2 n$ \dots $\log_2 n$

$$\Rightarrow \left(\frac{n}{2}\right) \log_2 n$$

$$\Rightarrow n \log_2 n$$

```
for ( i=0; i<n; i++)
```

```
  { i = i * k;
```

```
  }
```

$$\Rightarrow O(\log_k n)$$

```
value = 0;
```

```
for ( i=0; i<n; i++)
```

```
  {  $j=0; j<i$ 
    for ( j=0; j<i; j++)
```

```
      { value += 1;
```

```
      }
```

$i=0$ $i=1$ $i=2$ \dots $i=n-1$
 0 times 1 time 2 times \dots $n-1$ times

$$\Rightarrow 0 + 1 + 2 \dots n-1$$

$$\Rightarrow 1 + 2 + 3 \dots \textcircled{P}$$

$$= \frac{P(P+1)}{2}$$

$$\frac{(n-1)(n-1+1)}{2}$$

$$\Rightarrow \frac{(n-1)(n)}{2}$$

$$\Rightarrow 0((n)(n-1))$$