Greedy and Dynamic Programming

Dr. Gopal Chandra Jana

Assistant Professor

INTRODUCTION

- Greedy and Dynamic Programming are two powerful techniques in algorithm design.
- They are used to solve **optimization problems efficiently**.
- Understanding their principles and differences is crucial for tackling a wide range of computational problems.
- Also to consider certain parameters before applying any of these.

Choice Between Greedy and Dynamic Programming

- Greedy algorithms are preferred when the problem exhibits the **greedy-choice property** and **optimal substructure**.
- Dynamic Programming is suitable when the problem can be broken down into **overlapping sub-problems** with **optimal substructure**.
- Understanding the **problem's characteristics is essential** for selecting the appropriate technique.

Deciding whether to apply a greedy or dynamic programming approach to solve a problem depends on **several key parameters and characteristics of the problem**. Here are some factors to consider:

1. Optimal Substructure:

Greedy:

- Optimal solution can be constructed from locally optimal choices.
    - **Eg:** Finding the shortest path in a graph using **Dijkstra's algorithm**.

Dynamic Programming:

- Optimal solution can be derived from optimal solutions to sub-problems.
    - **Eg: Calculating the nth Fibonacci number using memoization.**

2. Overlapping Sub-problems:

Greedy:

- If the problem **does not have overlapping sub-problems**, a greedy approach is usually sufficient.
  - **Eg: Coin Change Problem where each coin can only be used once.**

Dynamic Programming:

- If the problem **can be broken down into overlapping sub-problems**, dynamic programming is more appropriate.
  - **Eg: Coin Change Problem where coins can be reused to make change.**

3. Greedy Choice Property:

Greedy:

- Makes locally optimal choices expecting a globally optimal solution.
  - **Eg: Fractional Knapsack Problem, where items can be divided and selected based on their value-to-weight ratio.**

Dynamic Programming:

- Evaluates all possible choices for optimal solution.
  - **Eg: 0/1 Knapsack Problem, where items cannot be divided, and a subset must be chosen to maximize value without exceeding the weight limit.**

4. Complexity and Efficiency:

Greedy:

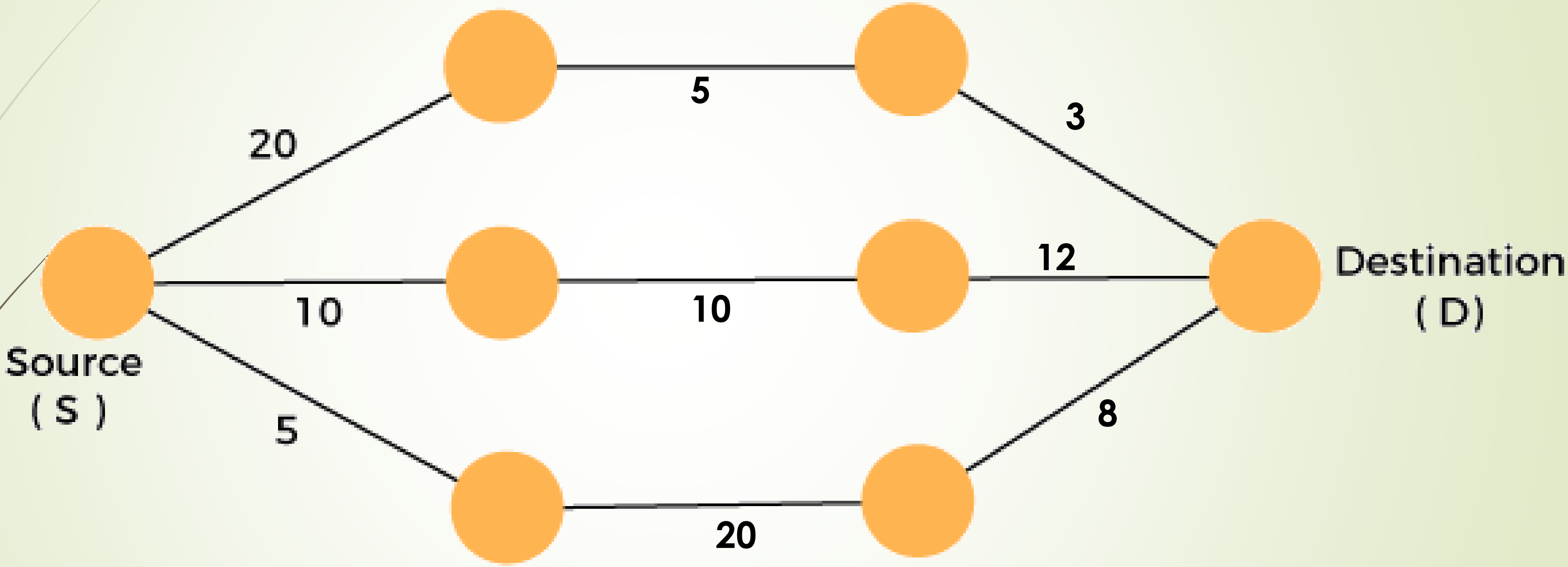
- Simple and efficient but **may not always provide optimal solution.**
  - **Eg: Activity Selection Problem, where activities with the earliest finish times are chosen iteratively.**
- Typically **requires less memory**.
  - **Eg: Huffman Coding for data compression, which constructs a binary tree based on the frequency of characters.**

Dynamic Programming:

- Dynamic programming can **guarantee the optimal solution** but may be more **computationally expensive**.
  - **Eg: Longest Common Subsequence Problem, where all possible subsequences must be examined to find the longest common subsequence.**
- May require **more memory due to storing solutions to sub-problems**.
  - **Eg: Matrix Chain Multiplication Problem, which involves finding the most efficient way to multiply matrices.**

Greedy Algorithm

- Greedy algorithms make decisions based on the **current best choice** without considering future consequences.
- They aim to find the globally optimal solution by making a series of locally optimal choices.
- Typically efficient and easy to implement.
- **Examples:** Dijkstra's algorithm for shortest paths
  Kruskal's algorithm for minimum spanning trees.


20
5
3
10
10
12
Destination
(D)
Source
(S)
5
20
8

Greedy Problems:

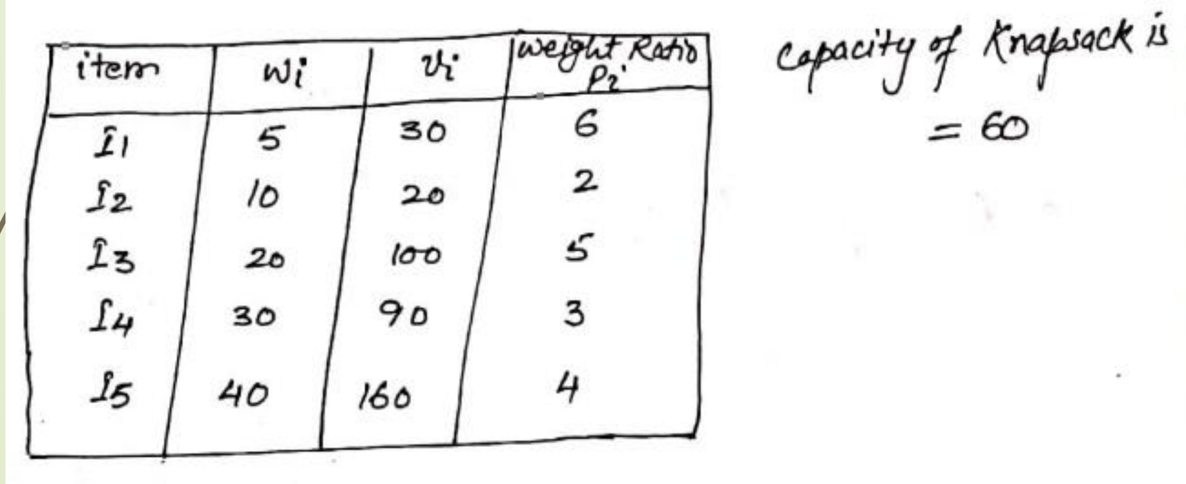
- **Fractional Knapsack Problem**: Given items with weight and value, determine the maximum value of fractions of the items that can be taken into a knapsack of limited weight capacity.
- **Activity Selection Problem**: Given a set of activities with start and finish times, select the maximum number of non-overlapping activities that can be performed by a single person.
- **Huffman Coding**: Given a set of characters and their frequencies, construct a binary tree such that the encoded binary codes for characters have minimum total length.
- **Coin Change Problem**: Given a set of coin denominations and a target amount, find the minimum number of coins needed to make up the target amount.
- **Job Sequencing Problem**: Given a set of jobs with deadlines and profits, find the maximum profit subset of jobs that can be completed within their deadlines.
- **Minimum Spanning Tree Algorithms**: Algorithms like Prim's and Kruskal's are greedy approaches to find the minimum spanning tree in a connected, undirected graph.
- **Dijkstra's Algorithm**: To find the shortest path between nodes in a graph with non-negative edge weights.

Fractional Knapsack

Given the weights and values of N items, in the form of {weight, Value} put these items in a knapsack of capacity W to get the maximum total profit in the knapsack. In Fractional Knapsack, we can break items for maximizing the total value of the knapsack.

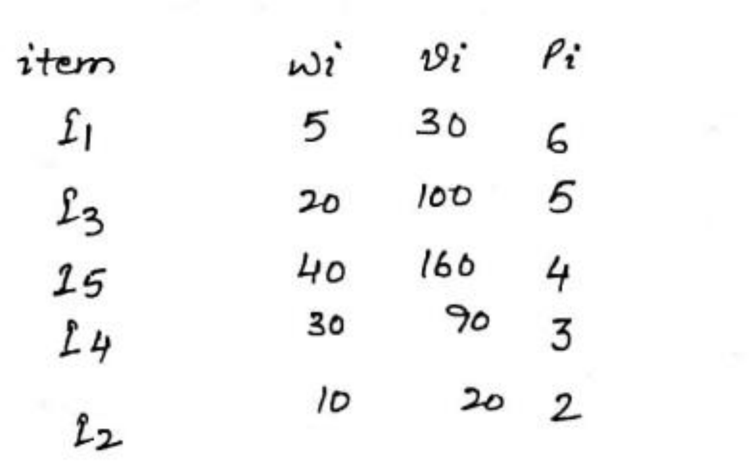
Input: arr[] = {{5, 30}, {10, 20}, {20, 100}, {30, 90}, {40, 160}}, W = 60

Output: 270



item	w_i	v_i	weight Ratio P_i
I_1	5	30	6
I_2	10	20	2
I_3	20	100	5
I_4	30	90	3
I_5	40	160	4

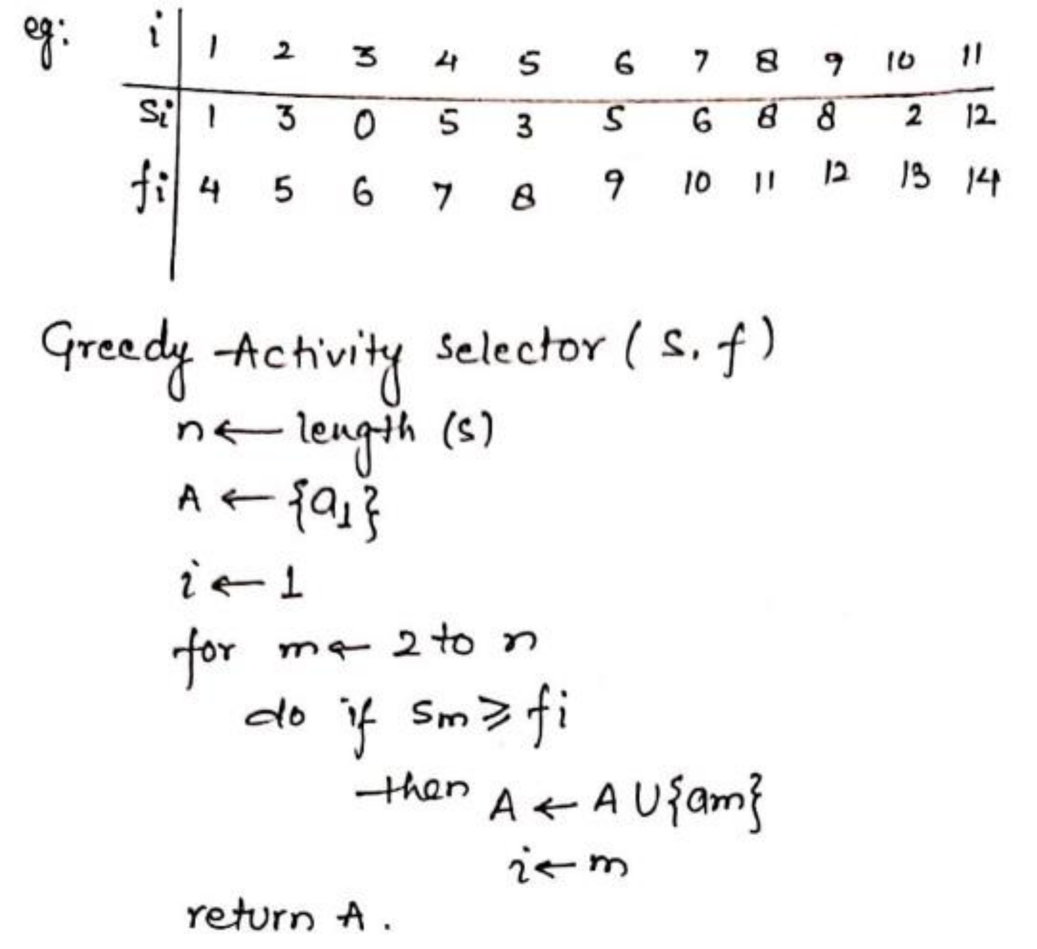
Capacity of Knapsack is = 60



item	w_i	v_i	P_i
I_1	5	30	6
I_3	20	100	5
I_5	40	160	4
I_4	30	90	3
I_2	10	20	2

Net Selected Value = I1 + I3 + I5*35/40 = 30+100+160*35/40 =270

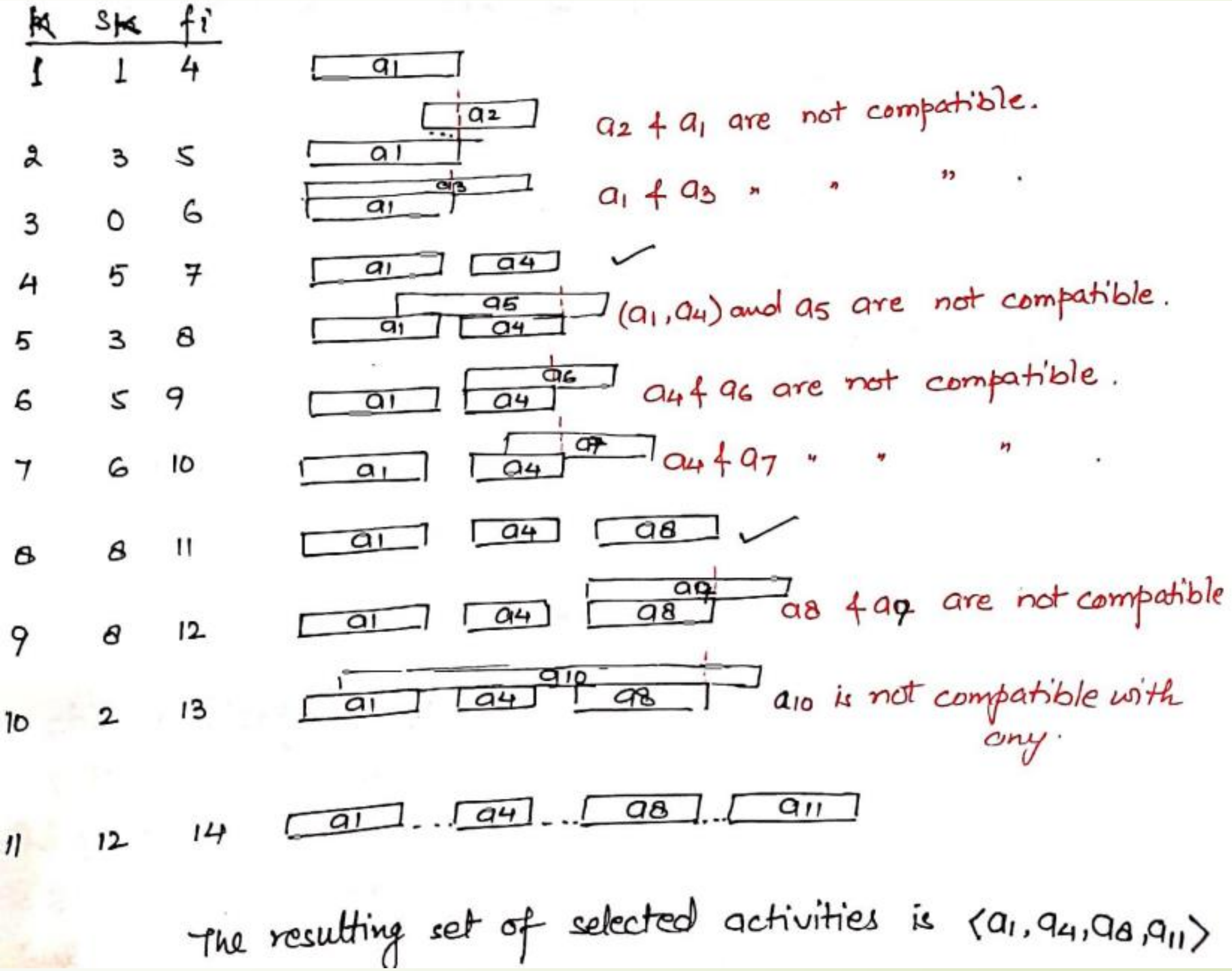
Activity Selection Problem



eg:

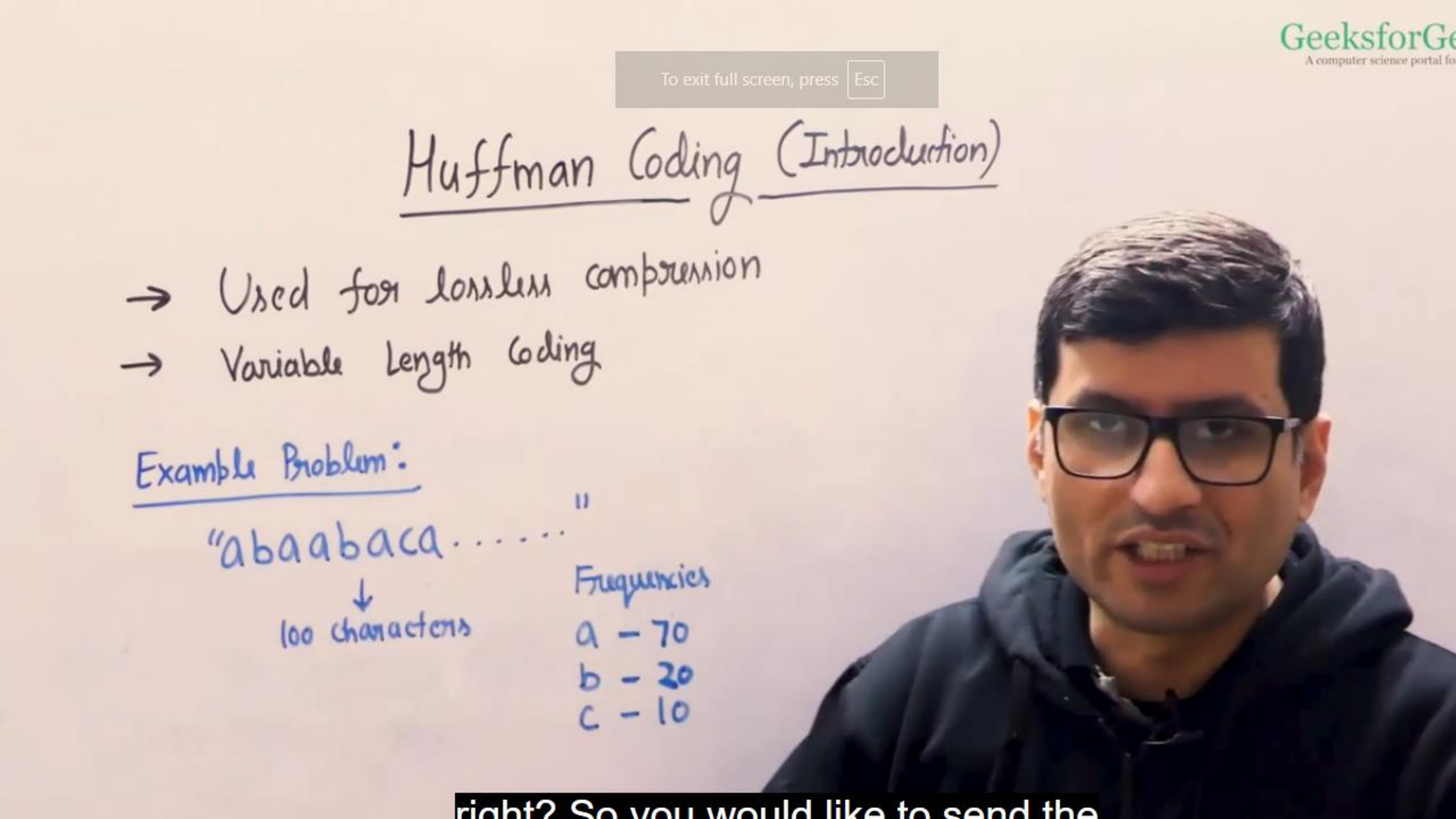
i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	8	9	10	11	12	13	14

```
Greedy-Activity selector (s, f)
    n ← length (s)
    A ← {a_1}
    i ← 1
    for m ← 2 to n
        do if s_m ≥ f_i
            then A ← A U {a_m}
                 i ← m
    return A.
```



k	s_k	f_i	
1	1	4	
2	3	5	a_2 & a_1 are not compatible.
3	0	6	a_1 & a_3 " " " .
4	5	7	✓
5	3	8	(a_1, a_4) and a_5 are not compatible.
6	5	9	a_4 & a_6 are not compatible.
7	6	10	a_4 & a_7 " " " .
8	8	11	✓
9	8	12	a_8 & a_9 are not compatible
10	2	13	a_{10} is not compatible with any.
11	12	14	

The resulting set of selected activities is $\langle a_1, a_4, a_8, a_{11} \rangle$



Huffman Coding (Introduction)

→ Used for lossless compression

→ Variable Length Coding

Example Problem:

"abaabaca......"

↓

100 characters

Frequencies

a - 70
b - 20
c - 10

right? So you would like to send the

Huffman Coding (Introduction)

- → Used for lossless compression
- → Variable Length Coding

Example Problem:

"ababaaca......"

↓

100 characters

Frequencies

a - 70 00
b - 20 01
c - 10 10

file. Can we do better than

Variable Length Huffman Coding

Greedy Idea:

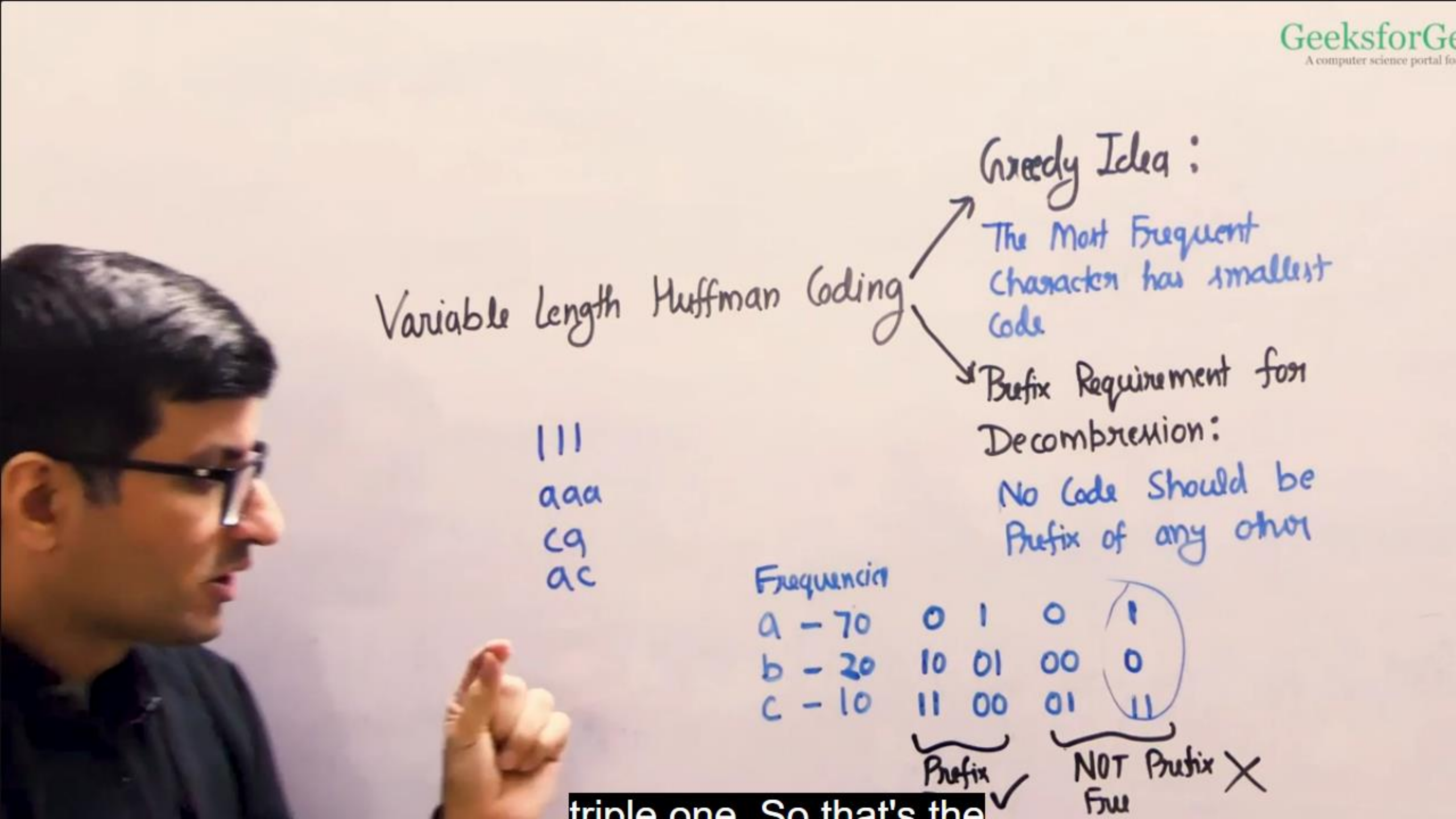
The Most Frequent Character has smallest code

Prefix Requirement for Decompression:

No Code Should be Prefix of any other

algorithm is the most frequent

Frequencies				
a - 70	0	1	0	1
b - 20	10	01	00	0
c - 10	11	00	01	11
	Prefix Free ✓		NOT Prefix Free ✗	



Variable Length Huffman Coding

Greedy Idea:

The Most Frequent Character has smallest code

Prefix Requirement for Decompression:

No Code Should be Prefix of any other

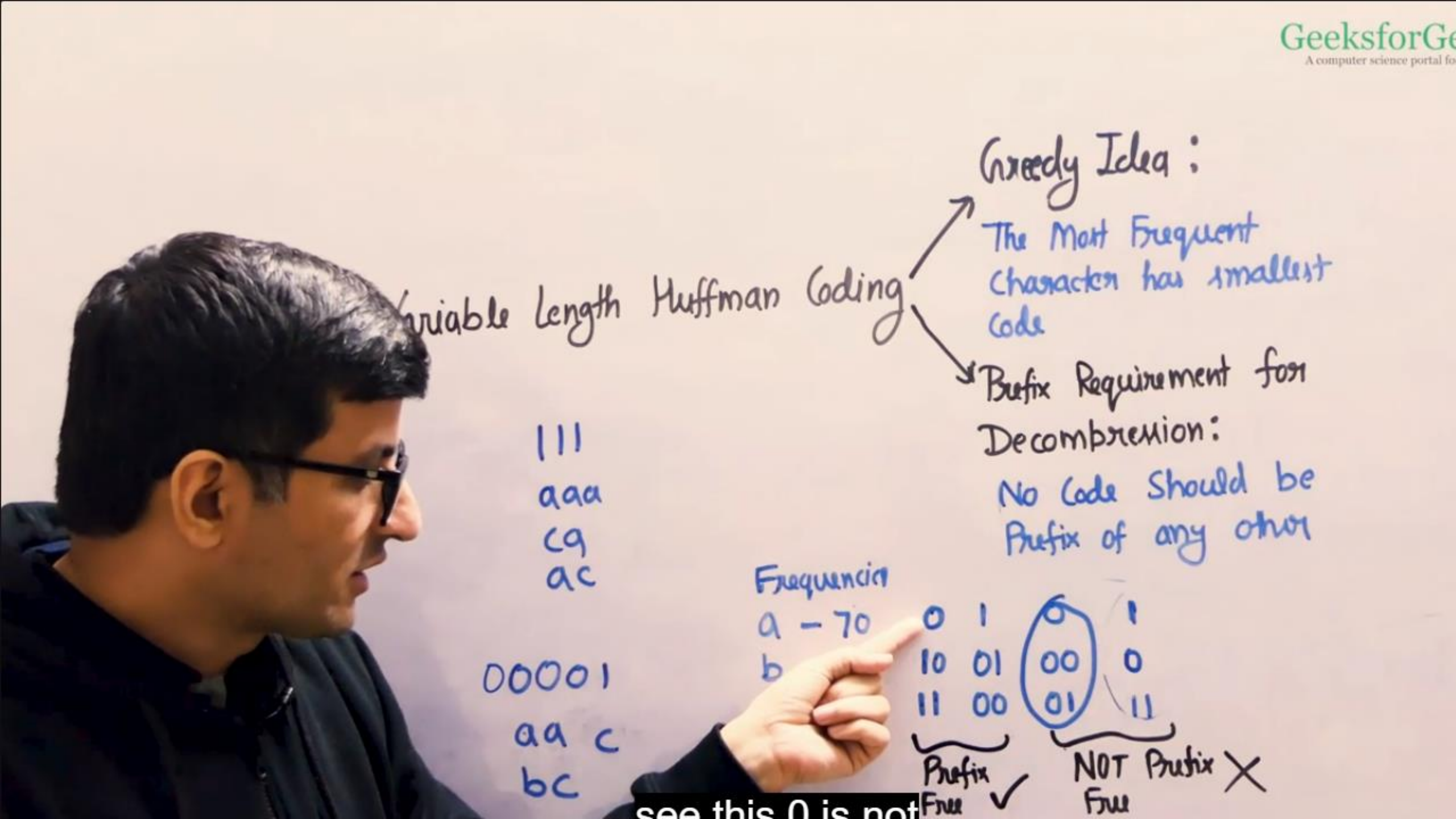
111
aaa
ca
ac

Frequencies				
a - 70	0	1	0	1
b - 20	10	01	00	0
c - 10	11	00	01	11

Prefix ✓

NOT Prefix Free ✗

triple one. So that's the



riable Length Huffman Coding

Greedy Idea:

The Most Frequent Character has smallest code

Prefix Requirement for Decompression:

No Code Should be Prefix of any other

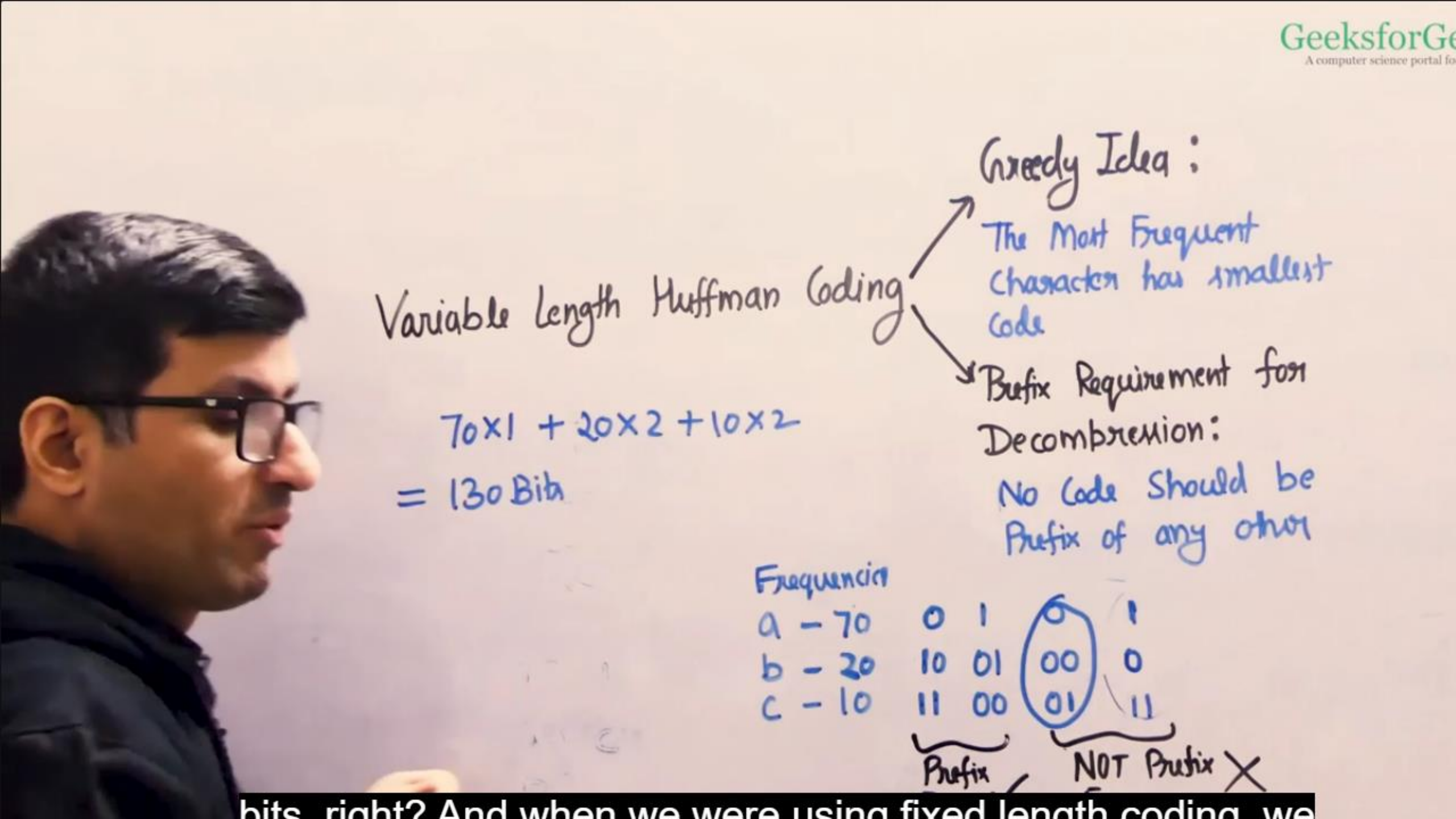
111
aaa
ca
ac

00001
aa c
bc

Frequencies
a - 70
b

0	1	0	1
10	01	00	0
11	00	01	11
Prefix Free ✓		NOT Prefix Free ✗	

see this 0 is not



Variable Length Huffman Coding

→ Greedy Idea:

The Most Frequent Character has smallest code

→ Prefix Requirement for Decompression:

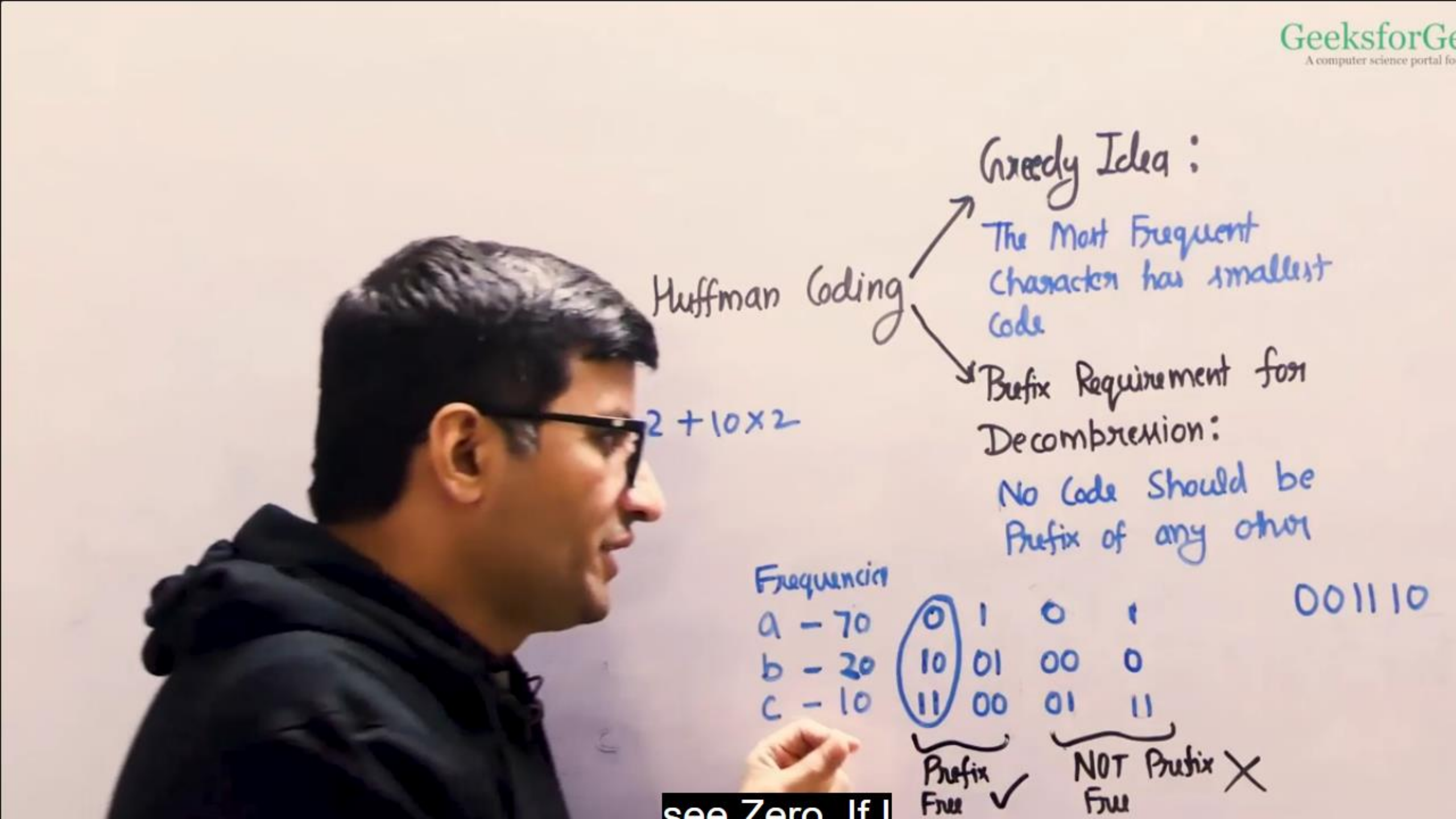
No Code Should be Prefix of any other

$70 \times 1 + 20 \times 2 + 10 \times 2$

$= 130$ Bits

Frequencies				
a - 70	0	1	0	1
b - 20	10	01	00	0
c - 10	11	00	01	11
	Prefix ✓		NOT Prefix ✗	

bits, right? And when we were using fixed length coding, we



Huffman Coding → Greedy Idea :

The Most Frequent Character has smallest code

Huffman Coding → Prefix Requirement for Decompression:

No Code Should be Prefix of any other

2 + 10 × 2

Frequencies				
a - 70	0	1	0	1
b - 20	10	01	00	0
c - 10	11	00	01	11
	Prefix Free ✓	NOT Prefix Free ✗		

001110

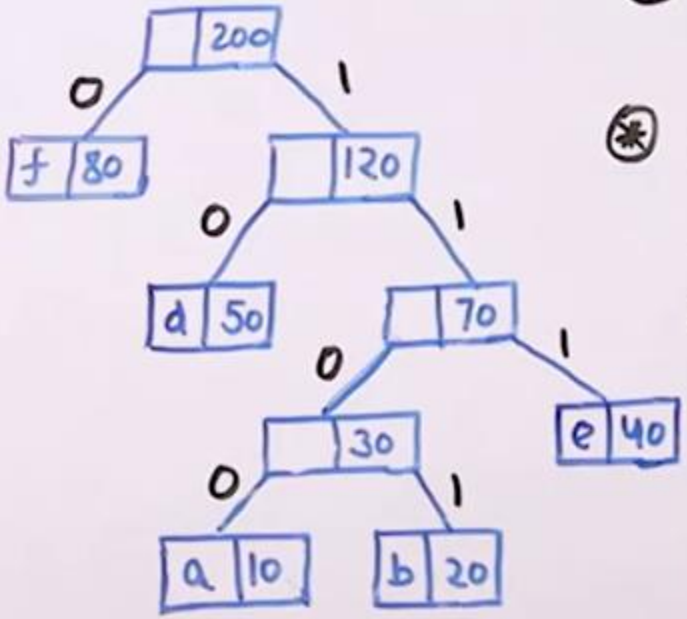
see Zero. If I

I/P : ['a', 'd', 'b', 'e', 'f']
[10, 50, 20, 40, 80]

Huffman Algorithm

(High level Idea)

1) Build a Binary Tree



- Every input character is a leaf
- Every left child edge is labelled as 0 and right edge as 1.
- Every root to leaf path represents Huffman code of the leaf.

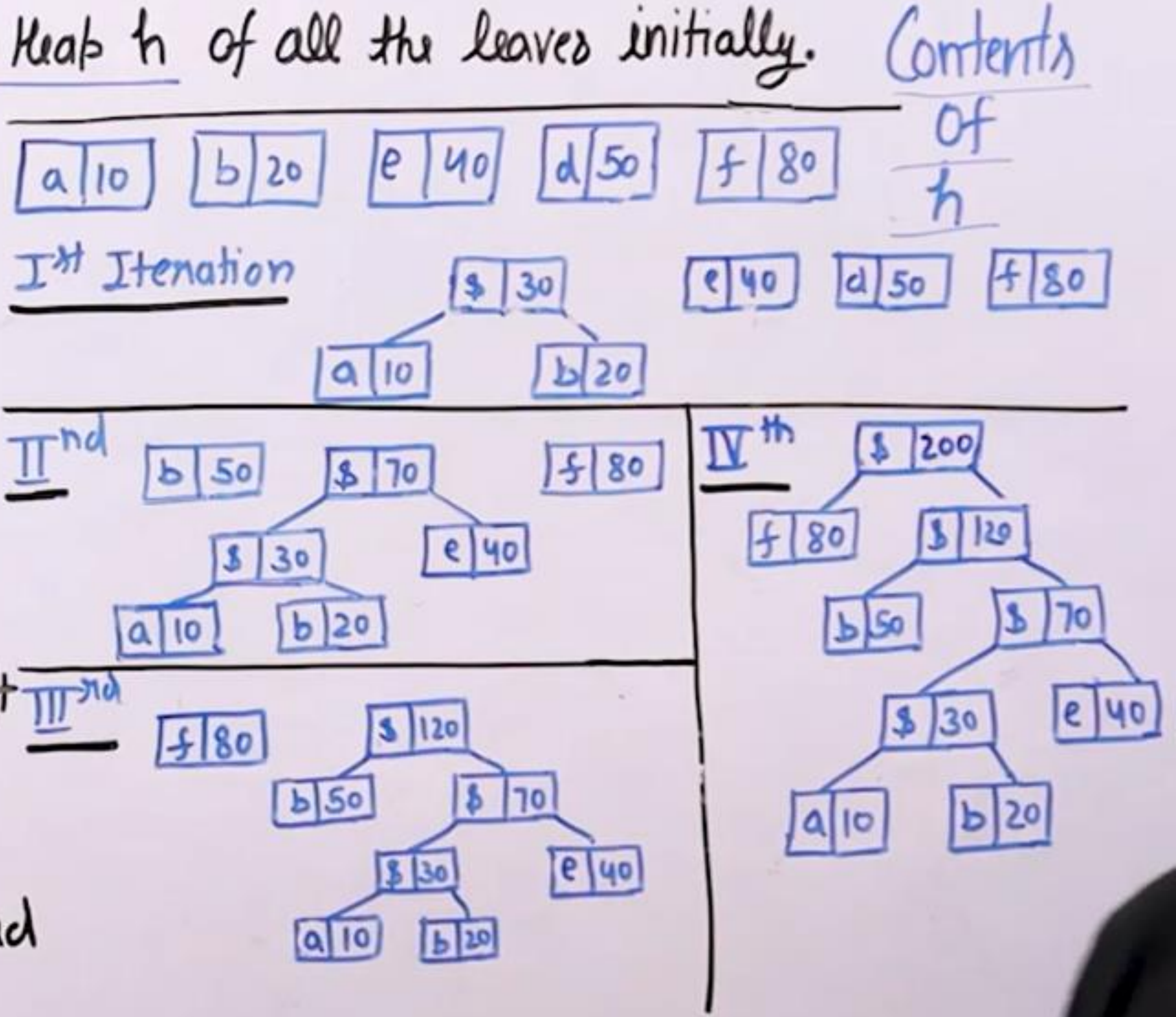
2) Traverse the Binary Tree and print the codes

f 0
d 10
a 1100

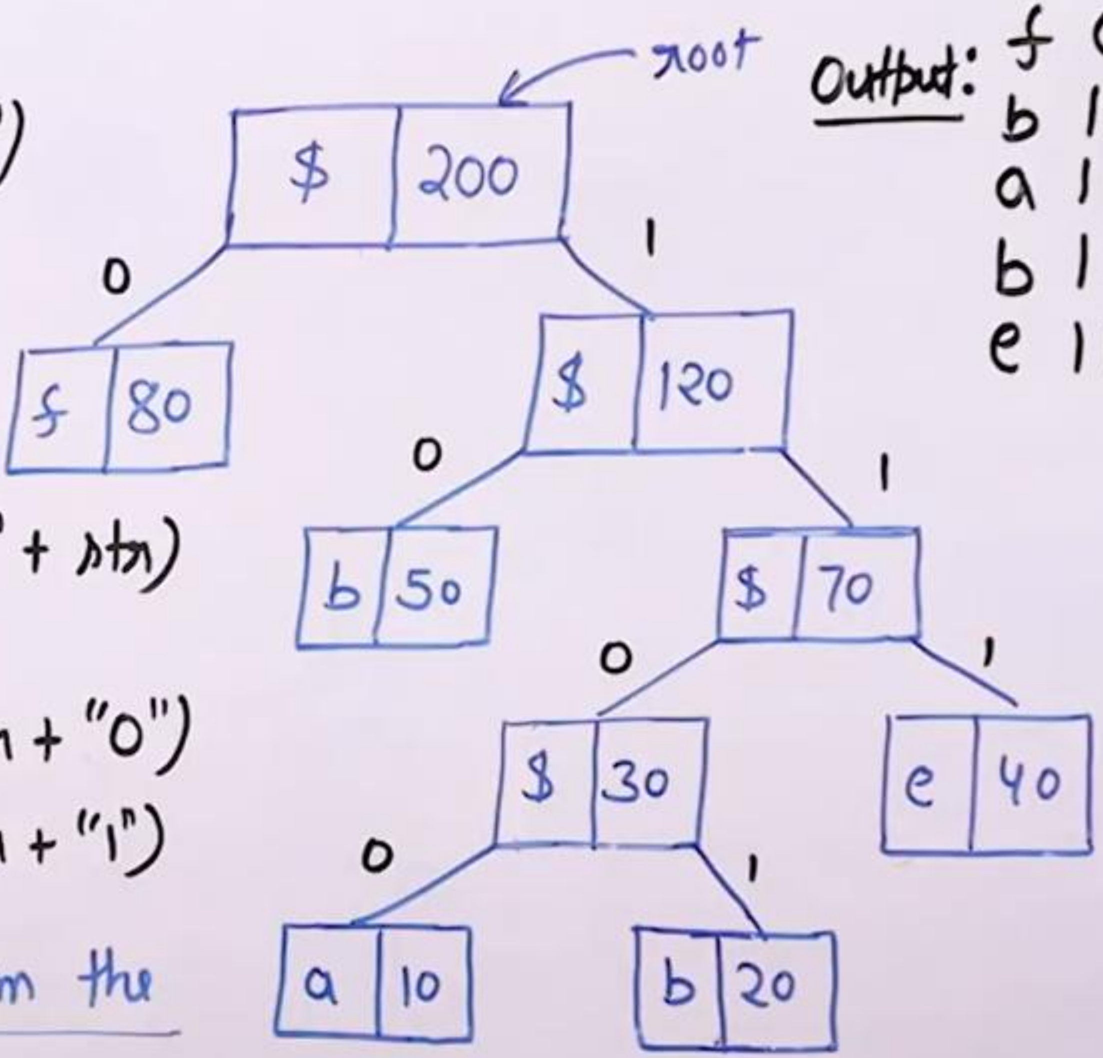
a 1100
b 1101
e 111

I/P: ['a', 'd', 'b', 'e', 'f']
[10, 50, 20, 40, 80]

```
1) Create leaf nodes and build a Min Heap h of all the leaves initially.
2) While h.size() > 1 :
    a) left = h.extractMin()
    b) right = h.extractMin()
    c) Create a new tree node with
        * character a $
        * Frequency as left.freq + right.freq
        * Left and right children as left
          and right respectively.
    d) Insert the new node into h
3) The only node left in h is our required
   Binary Tree.
```



```
void printCodes(root, str = "")
    if (root == null)
        return
    if (root.ch != '$')
        print(root.ch + " " + str)
        return
    printCodes(root.left, str + "0")
    printCodes(root.right, str + "1")
```



Output:

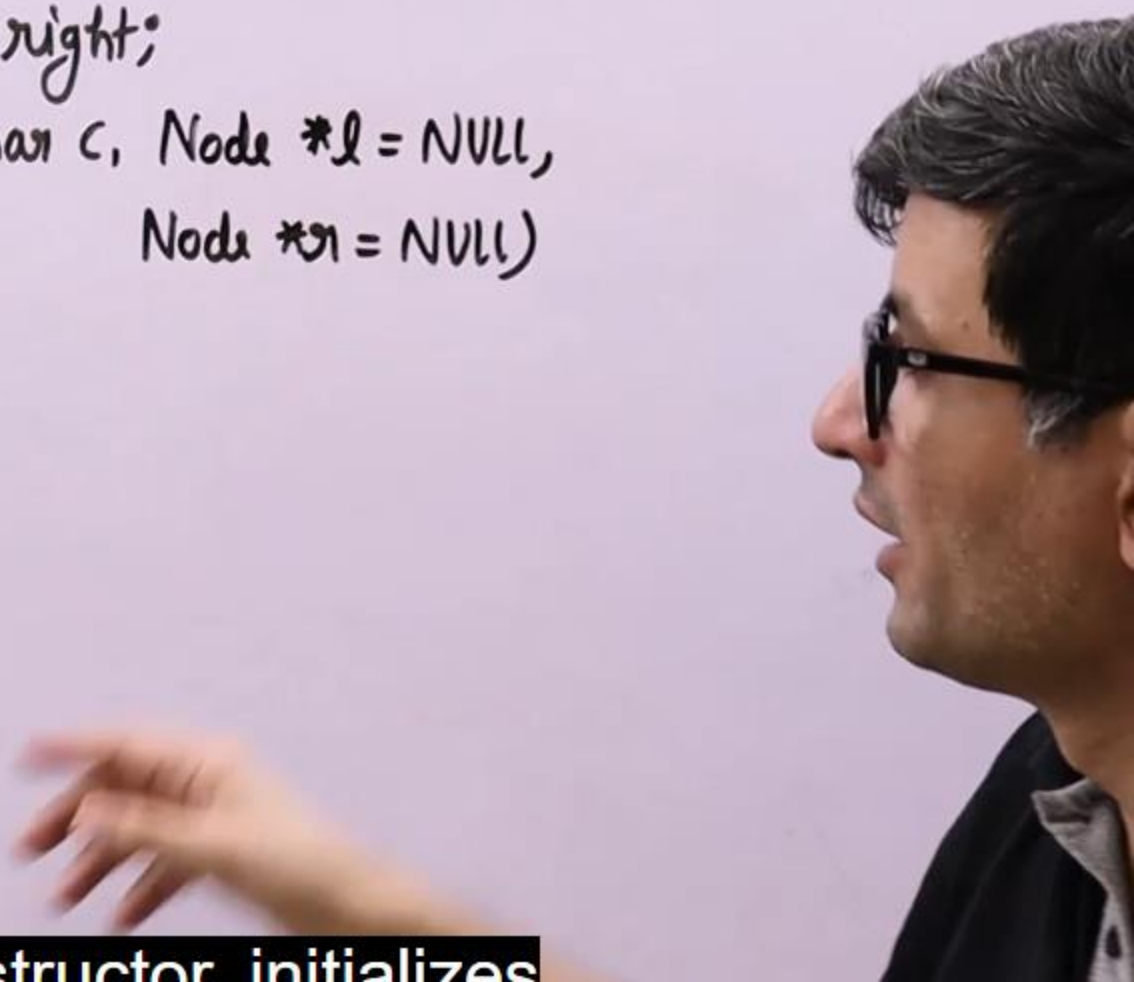
f 0
b 10
a 1100
b 1101
e 111

Printing Huffman Codes from the Built Binary Tree

C++ Implementation of Huffman Coding

Structure of a Binary (or Huffman) Tree Node.

```
struct Node
{ int freq;
  char ch;
  Node *left, *right;
  Node(int f, char c, Node *l = NULL,
                      Node *r = NULL)
  {
      freq = f;
      ch = c;
      left = l;
      right = r;
  }
};
```



right? So in the Constructor, initializes

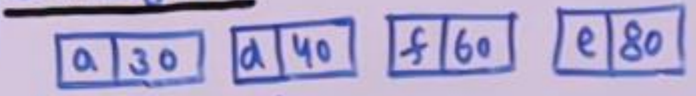
```
Void printHCodes(int arr[], int freq[], int n)
{ priority_queue<Node*, vector<Node*>, compare> h;
  for(int i=0; i<n; i++)
     h.push(new Node(arr[i], freq[i]));
  while(h.size() > 1)
  {
     Node *l = h.top(); h.pop();
     Node *r = h.top(); h.pop();
     Node *node = new Node('$', l->freq + r.freq, l, r);
     h.push(node);
  }
  printCodes(h.top(), "")
}
struct compare {
   bool operator()(Node *l, Node *r)
   { return l->freq < r->freq;
   }
}
```

```
void printCodes(Node *root, string str)
{   if(root->ch != '$')
    { cout << root->ch << " " << str << "\n";
      return;
    }
    printCodes(root->left, str + "0");
    printCodes(root->right, str + "1");
}
```

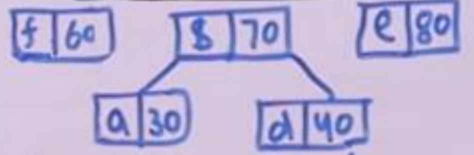
arr[] = {'a', 'd', 'e', 'f'}

freq[] = {30, 40, 80, 60}

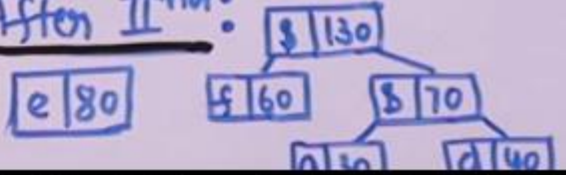
Initially h:



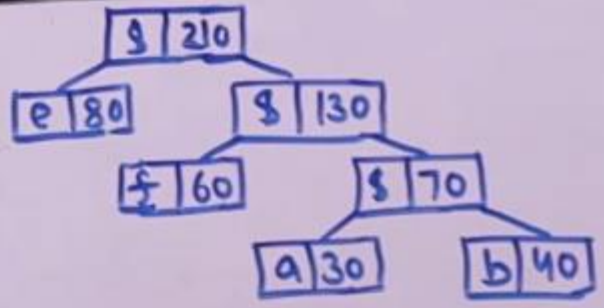
After Ist Iteration



After IInd:



After IIIrd



30 40 80, 60, right. This is the

Huffman DeCode

```
//Function to return the decoded string.
string decodeHuffmanData(struct MinHeapNode* root, string s)
{
    // Code here
    string ans = "";
    struct MinHeapNode* curr = root;
    for (int i = 0; i < s.size(); i++) {
        if (s[i] == '0')
            curr = curr->left;
        else
            curr = curr->right;

        // reached leaf node
        if (curr->left == NULL and curr->right == NULL) {
            ans += curr->data;
            curr = root;
        }
    }
    // cout<<ans<<endl;
    return ans + '\0';
}
```

Coin Change -Greedy

Given a value of V Rs and an infinite supply of each of the denominations {1, 2, 5, 10, 20, 50, 100, 500, 1000} valued coins/notes, The task is to find the minimum number of coins and/or notes needed to make the change?

Examples:

Input: V = 121

Output: 3

Explanation: We need a 100 Rs note, a 20 Rs note, and a 1 Rs coin.

1.Declare a vector that store the coins.
2. while n is greater than 0 iterate through greater to smaller coins:
    1. if n is greater than equal to 2000 than push 2000 into the vector and decrement its value from n.
    2. else if n is greater than equal to 500 than push 500 into the vector and decrement its value from n.
    3. And so on till the last coin using ladder if else.

Coin Change -Greedy

Given a value of V Rs and an infinite supply of each of the denominations {1, 2, 5, 10, 20, 50, 100, 500, 1000} valued coins/notes, The task is to find the minimum number of coins and/or notes needed to make the change?

Examples:

Input: V = 121

Output: 3

Explanation: We need a 100 Rs note, a 20 Rs note, and a 1 Rs coin.

- Sort the array of coins in decreasing order.
- Initialize **ans** vector as empty.
- Find the largest denomination that is smaller than **remaining amount** and while it is smaller than the **remaining amount**:
  - Add found denomination to **ans**. Subtract value of found denomination from **amount**.
- If amount becomes **0**, then print **ans**.

Coin Change -Greedy

```
// C++ program to find minimum
// number of denominations
#include <bits/stdc++.h>
using namespace std;

// All denominations of Indian Currency
int denomination[]
   = { 1, 2, 5, 10, 20, 50, 100, 500, 1000 };
int n = sizeof(denomination) /
          sizeof(denomination[0]);
```

```
void findMin(int V)
{
   sort(denomination, denomination + n);
    // Initialize result
   vector<int> ans;
    // Traverse through all denomination
   for (int i = n - 1; i >= 0; i--) {
        // Find denominations
       while (V >= denomination[i]) {
          V -= denomination[i];
          ans.push_back(denomination[i]);
       }
   }
   for (int i = 0; i < ans.size(); i++)
       cout << ans[i] << " ";
}
```

Any question?